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The effect of parameter estimation on phase II control chart performance in monitoring financial GARCH processes with contaminated data

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Abstract

The application of control charts for monitoring financial processes has received a greater focus after recent global crisis. The Generelized AutoRegressive Conditional Heteroskedasticity (GARCH) time series model is widely applied for modelling financial processes. Therefore, traditional Shewhart control chart is developed to monitor GARCH processes. There are some difficulties in financial surveillance especially in the retrospective phase one of which being the posibility of existing outliers in the samples data. For this aim, in this paper some methods were proposed to estimate the parameters of the GARCH model based on maximum likelihood and robust estimation procedures. Then, the performance of Phase II residual Shewhart control chart with estimated parameters was evaluated according to in-control Average Run Length in the presence of outliers. The Monte Carlo simulation study was applied to evaluate the proposed methods considering different numerical examples. Finally, the US Dollar/Iran Rial (USD/IRR) exchange rate was considered for monitoring in which the results showed that the control chart was more sensitive when the robust methods were applied in the estimation procedure.

Keywords: Financial surveillance, retrospective phase, GARCH model, robust estimation, foreign exchange rate

1- Introduction

Recent financial crisis reveals the necessity of new methods for detecting unnatural conditions as soon as possible. Horel and Snee (2009) discussed about the importance of more attention to statistical engineering rather than statistical science to help practitioners. They also encourage the application of control charts in monitoring financial processes with the aim of insightful view from the process. The control chart is a powerful tool in statistical process control which is vastly developed to monitor industrial processes (Woodall and Montgomery, 2014). In recent years, control charts have taken more attention for monitoring financial processes (Golosnoy, 2016).

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The application of control charts for monitoring financial processes needs some justifications. For example, the financial processes based on their nature should be modeled with more advanced time series models rather than industrial processes. One of these models is general autoregressive conditional heteroskedasticity (GARCH) time series model which can well define many financial processes (Garthoff et al. 2014). Therefore, most of researches in this area are performed with this model structure. Frisen (2008) classified the subject of financial surveillance by presenting required steps and adjustments.

The practical application of control chart starts with Phase I analysis which is known as retrospective phase (Jones-Farmer et al. 2014). Traditionally, this phase consists of estimating the parameters, designing control chart and detecting the out-of-control samples in historical data. These steps are iteratively repeated until accurate and precise estimated parameters for process and control chart are obtained. Then, this designed control chart is performed in phase II to monitor future observations. Alongside the vast development of Phase I analysis and Phase II control charts separately, recently, investigating the effect of parameter estimation (as a part of phase I analyses) is involved in the control chart performance in Phase II (Psarakis et al. 2014). In spite of extra researches in this subject for independent processes, there are few works for time dependent observations.

Adams and Tseng (1998) investigated the robustness of Shewhart, exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts with estimated parameters when there are errors in sample data for monitoring autoregressive (AR) and integrated moving average (IMA) processes. Apley (2002) performed a survey on the effect of model uncertainty on the performance of adjusted EWMA control chart for monitoring ARMA processes. They concluded that the minimum required sample size is related to the autocorrelation value. Chin and Apley (2008) examined the effect of different types of errors on the robustness of control charts for monitoring processes with ARMA time series model. Dasdemir et al. (2016) compared the performance of modified Shewhart control chart with different estimators for monitoring AR processes. They considered several examples which contain outliers in sample data.

To the best of our knowledge, there is no research on the effect of parameter estimation for monitoring financial GARCH processes. In this paper, first, some difficulties in Phase I analysis in monitoring financial processes are explained. One of these problems is the possibility of existing outliers in sample data. To tackle with the outliers, it is proposed to consider robust estimators to design control chart for monitoring GARCH processes. In addition, there is no paper in the literature of designing control charts to estimate the parameters of the GARCH model with robust methods. There are different approaches in the robust estimation procedures in which some of them are too complicated. For example, Muler and Yohai (2008) proposed the robust M-estimator for GARCH models. For practical simplicity, in this paper, a simple filtering procedure is proposed based on the confidence interval (CI) to reduce the effect of possible outliers in sample data. Then, the performance of Phase II residual Shewhart control chart with estimated parameters is evaluated for monitoring financial GARCH processes in the presence of outliers based on in-control average run length (ARL). The effect of different estimation methods on the performance of the control chart are compared in several numerical examples through simulation studies. Finally, the proposed methods are performed to estimate the GARCH model parameters for monitoring USD/IRR exchange rate as the main motivation of this research.

The rest of the paper is organized as follows: In the next section, Phase I analysis in monitoring financial processes is deliberated. Then, the GARCH model, four estimation methods and residual Shewhart control chart are explained in Section 3. In Section 4, the effect of estimation methods on the control chart performance are compared in different numerical examples through simulation studies based on descriptive statistics. Application of the proposed methods is illustrated through a real case corresponding to financial processes in Section 5. Concluding remarks are presented in the final section.

2- Retrospective analysis in financial surveillance

The main motivation of developing control chart to monitor financial processes is the power of this tool in detecting assignable causes. In this subject, the control chart is adjusted with some special features of financial processes. Indeed, some assumptions of industrial applications are different from financial processes. For example, in industrial processes, if an assignable cause(s) occurs and the control chart signals, the practitioner can stop the machine and implement corrective action(s). While, financial processes, in most of the cases, based on their nature perform continuously and they cannot be stopped or changed easily by corrective action(s). Hence, expert can only recognize the behavior of the process using statistical methods and can make a proper decision in due time. Therefore, defining stable conditions to gather in-control data for Phase I analysis is rather impossible in some cases. In the other words, performing Phase I analyses faces with the high probability of existing outliers in sample data. For example, Herwarts and Reimers (2002) pointed out the problem of defining stable financial target process in the monetary policies of the US and Japan foreign exchange rates. Correspondingly, Garthoff et al. (2014) stated "Phase I cannot be clearly defined regarding financial time series".

Furthermore, a basic inference in financial data is the nature of time dependency. According to this feature, one sample could not easily be neglected in Phase I analysis (like the simple independent observations). Eliminating one sample in time series leads to complexity in estimator. Thus, in spite of vast development of control charts in Phase I analysis of time independent processes, there are few works for time dependent processes (Boyles, 2000). In time series control charts, usually retrospective analyses have been limited to the effect of parameter estimation. Form the other side; financial experts believe that each observation contains information. Therefore, they recommend modeling the financial processes based on the maximum possible samples. Usually, the parameters of the model are estimated based on the large size of in-control sample data.

In this paper, time dependent GARCH process is considered to define financial process. In addition to traditional maximum likelihood estimation (MLE), three estimation methods are proposed based on robust M-estimator and the filtering procedure to estimate the parameters of the model. Then, phase II residual Shewhart control chart is designed to monitor financial GARCH processes. To evaluate the effect of parameter estimation on the performance of Phase II financial control chart, the large sizes of sample data are generated with different rates of outlier based on Monte Carlo simulation. Hence, the effect of estimation methods are compared under different numerical examples based on the in-control ARL. The final aim is producing some adjustments to improve the performance of control chart in the presence of outliers. In the most of researches in this area, the average, median and standard deviation of in-control ARL are considered as criteria. It is desired to have the average and median of ARL close to a predefined value, while the minimum value of standard deviation is required.

3- Control chart design

In the previous section, the problem of retrospective phase I analyses in financial processes is defined generally. In this section, the GARCH model is defined as the most popular model in financial processes. The reason of this selection is to model USD/IRR exchange rate as the main motivation of this research. Then, four methods are explained to estimate the parameters of the GARCH model. Finally, the residual Shewhart control chart is described for performance analyses of Phase II control chart.

3-1- GARCH model

Engle (1982) presented ARCH model to define volatility in conditional variance mode. Then, Engle and Bollerslev (1986) developed GARCH model as a natural development of AR models to ARMA ones. The GARCH (*p*,*q*) model is defined as Equation (1).

$$x_t = \varepsilon_t \sqrt{h_t}, \tag{1}$$

For $t = 1, 2, ..., m$ in which ε_t is innovation and h_t is defined as Equation (2).

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}. \tag{2}$$

In this paper, it is assumed that innovation independently and identically follows standard normal distribution. $\boldsymbol{\theta} = (\omega, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p)^T$ is the vector of parameters and has the constraints of $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. Financial data in the most of cases are well explained by GARCH model. In the following, four methods are explained to estimate the parameters of the financial GARCH processes. Note that the residuals of the model (ε_t) is computed based on Equation (3).

$$\varepsilon_t = \frac{x_t}{\sqrt{h_t}}. \tag{3}$$

3-2- Maximum likelihood estimation (MLE)

As the first method, the maximum likelihood estimation is considered as the most usual method to estimate the parameters of the GARCH model. In this method, if $(\varepsilon_1, \ldots, \varepsilon_m)$ is defined as the residuals of length *m*, the likelihood function is calculated as Equation (4). It is usual to maximize the natural logarithm of likelihood function. The simplified log likelihood function is presented in Equations (5) as expressed in Engle and Bollerslev (1986).

$$L(\boldsymbol{\theta}) = L(\boldsymbol{\theta}; \varepsilon_1, \ldots, \varepsilon_m) = \prod_{t=1}^{m} \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{\varepsilon_t^2}{2h_t}\right). \tag{4}$$

$$I(\boldsymbol{\theta}) = \log L(\boldsymbol{\theta}) = -\frac{m}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{m}\left(\log(h_t) + \varepsilon^2 h_t^{-1}\right), \tag{5}$$

Note that the initial values $\varepsilon_0, \ldots, \varepsilon_{1-q}, h_0^2, \ldots, h_{1-p}^2$ should be predefined or initiated. Then, $\boldsymbol{\theta}$ is estimated such that $\hat{\boldsymbol{\theta}}$ maximizes log likelihood function in Equations (6).

$$\hat{\boldsymbol{\theta}} = \arg\max I(\boldsymbol{\theta}). \tag{6}$$

Equation (6) is the general form of MLE method (Scholz, 2006). Bollerslev (1986) calculated partial derivatives of the log likelihood model to obtain MLE of parameters. There are also software packages such as MATLAB and R to estimate the parameters of the GARCH model based on MLE method.

3-3- Robust M-estimation (RME)

In the second method, using the robust M-estimator (Muler and Yohai, 2008) is proposed to design control chart. This method performs based on the quasi maximum likelihood function (Berkes et al., 2003) for GARCH model which can be written as Equation (7).

$$M(\boldsymbol{\theta}) = \frac{1}{m-p}\sum_{t=p+1}^{m} \rho(w). \tag{7}$$

In this function, $\rho(w) = -\log(g(w))$ and $w = \log(x_t^2) - \log(h_t)$. If $(\varepsilon_1, \ldots, \varepsilon_m)$ identically follow independent standard normal distribution, $g(w)$ can be written as Equation (8).

$$g(w)=\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{e^{w}-w}{2}\right)}. \tag{8}$$

Muler and Yohai (2008) showed that if the ρ function is replaced with $\rho*$ in Equation (9), the robust M-estimator for the parameters of the GARCH model (**θ**) is defined as Equation (10).

$$\rho^*=\begin{cases}\rho & if \quad \rho\le 4\\ P(\rho) & if \quad 4<\rho\le 4.3,\\ 4.15 & if \quad \rho>4.3\end{cases} \tag{9}$$

$$\hat{\boldsymbol{\theta}}=\arg\min M\left(\boldsymbol{\theta}\right). \tag{10}$$

The function of $P(\rho)$ in Equation (9) is a polynomial trend between a=4 and b=4.3, and defined as Equation (11).

$$P(\rho)=\frac{2}{(b-a)^{3}}\left(\frac{1}{4}\left(\rho^{4}-a^{4}\right)-\frac{1}{3}(2a+b)\left(\rho^{3}-a^{3}\right)+\frac{1}{2}\left(a^{2}+2ab\right)\left(\rho^{2}-a^{2}\right)\right)+ \\ -\frac{2a^{2}b}{(b-a)^{3}}(\rho-a)-\frac{1}{3(b-a)^{2}}(\rho-a)^{3}+\rho. \tag{11}$$

Muler and Yohai (2008) showed that this method performs well in comparison with the other estimation methods.

3-4- Filtered maximum likelihood estimation (FMLE)

It is well known that the financial indices are affected by many parameters in the real side of economy. Therefore, diagnosing the outliers or defining a real assignable cause for an outlier is rather impossible. Furthermore, searching for the out-of-control samples and finding the source of variation in a long period of the past data are not cost efficient procedures. Moreover, the complexity of the robust methods leads the practitioners to the traditional simple methods. Therefore, in the third estimation method, it is proposed to filter the residuals based on the presumed confidence interval.

Let define ε_t as the residual of the model. Since, the residuals of the model independently follow identical standard normal distribution, two sided (1-α) percent confidence interval (CI) is defined as $[z_{1-\alpha/2}, z_{\alpha/2}]$. Hence, the residuals which are outside of this CI are simply omitted in the estimation procedure regardless to their cause. If the parameters of the model are re-estimated based on the remained data, this filtering procedure could lead to the robust result in the next estimation. It should be noted that some software packages can easily deal with not available (N.A.) samples in MLE method. Moreover, the similar filtering procedure was performed by Grossi and Morelli (2006) and Carnero et al. (2008). Although this method has a weakness in possibly eliminating the common cause samples. However, it guarantees the robustness of the estimator in the presence of outliers. Therefore, as can be seen in the result of the next section, we recommend this method only in cases with high percent of the outliers. Finally, this filtering procedure continuously is repeated until there are no outliers in sample data. Accordingly, the following steps are proposed to estimate the parameters of the financial GARCH processes.

Do

a. Estimate the parameters of the model based on maximum likelihood estimation
b. Filter the residuals of the model based on (1-α)% CI

Repeat until there is no outlier in sample data

3-5- Filtered robust M-estimation (FRME)

In the fourth method, the same filtering procedure in the previous method (FMLE) is proposed, this time, with the robust M-estimator (Muler and Yohai, 2008). Therefore, the following steps are proposed to estimate the parameters of the financial GARCH processes.

Do

a. Estimate the parameters of the model using M-estimator (Muler and Yohai, 2008)
b. Filter the residuals of the model based on (1-α)% CI

Repeat until there is no outlier in sample data

3-6- Phase II residual Shewhart control chart

After estimating the parameters of the GARCH model, the residual Shewhart control chart (Severin and Schmid, 1998) is applied to monitor the process in Phase II. The control statistic is the residual of the model (ε_t) in Equation (3). Therefore, the symmetric control limits (UCL and LCL) are determined based on the standard normal distribution. The process is considered in-control until both conditions in Equation (12) are satisfied, simultaneously.

$$\begin{aligned} \varepsilon_t &\le UCL, \\ \varepsilon_t &\ge LCL. \end{aligned} \tag{12}$$

Usually, the control limits are determined such that the in-control ARL in Phase II equals to the predefined value. Traditionally, in-control ARL is considered equal to 370 in industrial applications, while the desired in-control ARL in financial applications is commonly set as 60, 120 and 240. When the process goes to the out-of-control state, the normal distribution is violated and control chart signals. In the next section, the performance of the residual Shewhart control chart with the estimated parameters is evaluated based on descriptive statistics of the in-control ARL.

4- Simulation studies

In this section, the performance of Shewhart control chart with estimated parameters is evaluated through simulation studies. Without loss of generality, the parameters (ω, α_1, β_1) in GARCH (1,1) model is considered equal to (0.4, 0.3, 0.3) . The outliers are involved in simulation studies as Equation (13).

$$x_t = \begin{cases} x_t & if \quad nu \ge r \\ Sx_t & if \quad nu < r \end{cases}, \tag{13}$$

Where *nu* is a random data generated from uniform distribution in the range of [0,1], *r* and *S* are the rate and the size of nuisance in data, respectively. This equation is the reformulation of volatility outlier (VO) in GARCH process by Hotta and Tsay (2012). The extraordinary shock of a sample in this formulation is considered as the *S* multiplied by the same sample.

Two examples are considered in this section. In these examples, the in-control ARLs are set equal to 120 and 370, respectively. For sensitivity analyses, two other ARL values equals to 60 and 240 are studied as well. The results of sensitivity analyses are not reported in this paper and available upon request. Because the similar results are obtained in these cases (in-control ARL equals to 60 and 240) and confirms the results in the cases of in-control ARL equals to 120 and 370.

In each simulation run, the parameters of the model are estimated based on 5000 samples using MLE, RME, FMLE and FRME methods. Then, in-control ARL, the average number of filtering iterations (itr) and the average number of total filtered samples (nfs) are obtained through 5000 replications. Finally, these steps are repeated 100 times to obtain descriptive statistics including mean, median and standard deviation for each mentioned criterion. For sensitivity analysis, the *r* and *S* values are changed in the range of (0, 0.02, 0.05, 0.1, 0.25) and (1.5, 3), respectively. The final aim is to find the proper method under different situations such that the average and median of in-control

ARL and estimated parameters close to the corresponding expected values with minimum standard deviation.

4-1- Example I

In this example, MLE, RME, FMLE and FRME methods are applied to estimate the parameters of GARCH(1,1) process in the presence of different rates of nuisance. CI in the filtering procedure is set based on 99.17 percent confidence level. Then, the performance of the residual Shewhart control chart with estimated parameters is evaluated based on the in-control ARL criterion. Note that the control limits (UCL and LCL) for the residual Shewhart control chart in Phase II is set equal to ±2.6383 for predefined in-control ARL equals to 120. Table 1 shows mean, median and standard deviation of the in-control ARL, the parameters of the GARCH model, the average number of filtering iterations and the average number of filtered samples when the size of nuisance (*S*) is set equal to 1.5. The highlighted columns in this table mark the first two best methods and the bold data indicate the best results.

Table 1. Descriptive statistics of the results when S is set equal to 1.5 for the first example

Statistic		Mean				Median				Standard Deviation			
Method		MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME
r	Criteria												
0	ARL	120.45	**119.34**	92.943	98.013	**120.57**	120.60	92.590	98.529	9.8118	9.5238	8.1947	8.9954
	ω	0.3962	**0.3972**	0.3679	0.3773	**0.3989**	0.3971	0.3726	0.3768	0.0341	0.0363	0.0391	0.0387
	α_1	0.3012	**0.3003**	0.2822	0.2851	**0.3043**	0.2992	0.2812	0.2854	0.0263	0.0247	0.0291	0.0272
	β_1	0.3045	**0.3026**	0.3063	0.3020	**0.3053**	0.3029	0.3057	0.2996	0.0465	0.0463	0.0552	0.0523
	itr			5.3600	4.8100			5.0000	5.0000			1.4321	1.3310
	nfs			69.190	65.720			68.000	64.000			12.953	12.106
0.02	ARL	129.65	**125.53**	96.323	102.34	129.88	**123.06**	96.027	102.02	10.228	11.078	7.9529	9.5706
	ω	0.4164	**0.4043**	0.3848	0.3821	0.4175	**0.4015**	0.3773	0.3797	0.0386	0.0389	0.0428	0.0462
	α_1	0.3022	**0.2987**	0.2802	0.2855	0.3032	**0.2984**	0.2798	0.2873	0.0249	0.0269	0.0283	0.0303
	β_1	0.2938	**0.3044**	0.2947	0.3039	0.2947	**0.3006**	0.3000	0.2989	0.0455	0.0466	0.0554	0.0620
	itr			5.4700	4.6400			5.0000	5.0000			1.6296	1.1328
	nfs			74.560	69.480			74.000	69.000			11.761	11.513
0.05	ARL	143.14	137.43	101.98	**108.72**	142.96	136.08	102.81	**107.66**	12.514	12.347	8.6808	10.323
	ω	0.4307	0.4382	0.3852	**0.4089**	0.4344	0.4368	0.3883	**0.4054**	0.0418	0.0430	0.0467	0.0497
	α_1	0.2990	0.2952	0.2735	**0.2828**	0.2946	0.2920	0.2719	**0.2821**	0.0271	0.0256	0.0296	0.0275
	β_1	0.2972	0.2848	0.3091	**0.2856**	0.2924	0.2910	0.3110	**0.2905**	0.0517	0.0500	0.0669	0.0621
	itr			5.4100	**4.9500**			5.0000	**5.0000**			1.4914	1.4521
	nfs			79.280	**77.890**			79.000	**77.000**			10.572	13.250
0.1	ARL	171.32	153.95	112.30	**119.20**	170.03	154.98	111.44	**120.42**	16.554	13.642	10.202	10.884
	ω	0.4662	0.4539	0.4139	**0.4159**	0.4650	0.4499	0.4183	**0.4080**	0.0411	0.0452	0.0484	0.0472
	α_1	0.3018	0.2881	0.2787	**0.2750**	0.3017	0.2848	0.2785	**0.2752**	0.0296	0.0237	0.0289	0.0260
	β_1	0.2883	0.2928	0.2892	**0.3000**	0.2906	0.2897	0.2826	**0.3025**	0.0468	0.0467	0.0574	0.0566
	itr			5.4000	**5.1100**			5.0000	**5.0000**			1.4284	1.4764
	nfs			91.310	**87.250**			90.500	**85.500**			12.103	13.478
0.25	ARL	278.11	237.95	**156.82**	169.43	276.29	231.83	**157.01**	167.55	29.417	28.816	16.860	19.844
	ω	0.5455	0.5329	**0.4639**	0.4739	0.5400	0.5292	**0.4706**	0.4744	0.0524	0.0518	0.0621	0.0524
	α_1	0.2929	0.2773	**0.2636**	0.2681	0.2894	0.2749	**0.2661**	0.2682	0.0299	0.0295	0.0274	0.0294
	β_1	0.2962	0.2945	**0.3044**	0.3035	0.2922	0.2943	**0.2976**	0.3087	0.0496	0.0527	0.0693	0.0609
	itr			**6.1200**	4.9700			**6.0000**	5.0000			1.8818	1.2099
	nfs			**111.23**	107.81			**111.00**	108.00			14.980	12.917

The results of Table 1 show that when the rate of nuisance (*r*) in samples is very small, here equal or less than 0.02, RME method performs well and MLE method is the second proper method. Indeed, this reveals the weakness of filtering procedure in the absence of outliers. The reason of this weakness is enforcement of filtering procedure for reducing the volatility of the process when it is not necessary. When *r* increases to 0.05, FRME and RME methods are the first and second appropriate methods. In this situation, the robust procedure could well overcome the nuisance in both RME and FRME methods. Afterwards, when *r* becomes equal to 0.1, FRME and FMLE are the first and second well methods. As *r* increases, the performance of the filtering procedure improves and gets better than the robust procedure. This shows the robustness of the filtering procedure under large rates of outliers. Table 2 shows the same results when the size of nuisance (*S*) is set equal to 3.

Table 2. Descriptive statistics of the results when S is set equal to 3 for the first example

Statistic		Mean				Median				Standard Deviation			
Method		MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME
r	Criteria												
0	ARL	120.45	**119.34**	92.943	98.013	**120.57**	120.60	92.590	98.529	9.8118	9.5238	8.1947	8.9954
	ω	0.3962	**0.3972**	0.3679	0.3773	**0.3989**	0.3971	0.3726	0.3768	0.0341	0.0363	0.0391	0.0387
	α_1	0.3012	**0.3003**	0.2822	0.2851	**0.3043**	0.2992	0.2812	0.2854	0.0263	0.0247	0.0291	0.0272
	β_1	0.3045	**0.3026**	0.3063	0.3020	**0.3053**	0.3029	0.3057	0.2996	0.0465	0.0463	0.0552	0.0523
	itr			5.3600	4.8100			5.0000	5.0000			1.4321	1.3310
	nfs			69.190	65.720			68.000	64.000			12.953	12.106
0.02	ARL	203.40	**127.70**	97.489	103.87	200.71	**126.59**	96.594	101.98	29.186	13.095	8.8907	10.020
	ω	0.5187	0.4602	0.3877	**0.3986**	0.5149	0.4603	0.3822	**0.3903**	0.0665	0.0508	0.0479	0.0463
	α_1	0.3102	0.2808	0.2722	**0.2838**	0.3108	0.2805	0.2739	**0.2814**	0.0418	0.0315	0.0268	0.0316
	β_1	0.2597	0.2596	0.2988	**0.2888**	0.2595	0.2634	0.3015	**0.2915**	0.0721	0.0580	0.0597	0.0598
	itr			5.8200	4.9100			6.0000	5.0000			1.6167	1.4220
	nfs			104.80	99.380			105.50	98.000			14.129	11.839
0.05	ARL	426.48	142.58	105.88	**112.06**	420.29	142.21	105.84	**111.36**	73.703	14.425	9.7056	10.159
	ω	0.6785	0.5176	0.4043	**0.4094**	0.6689	0.5235	0.4062	**0.4110**	0.1012	0.0624	0.0536	0.0457
	α_1	0.3137	0.2483	0.2676	**0.2723**	0.3045	0.2456	0.2652	**0.2703**	0.0499	0.0298	0.0302	0.0305
	β_1	0.2393	0.2453	0.2981	**0.2985**	0.2317	0.2414	0.3007	**0.2906**	0.0897	0.0652	0.0697	0.0577
	itr			6.3500	**5.0200**			6.0000	**5.0000**			1.4933	1.3407
	nfs			155.22	**152.67**			154.50	**153.00**			16.586	14.618
0.1	ARL	1128.2	173.58	**124.98**	128.16	1067.6	171.81	**123.51**	126.81	284.55	23.673	14.320	14.642
	ω	0.9583	0.6536	**0.4352**	0.4699	0.9641	0.6493	**0.4330**	0.4658	0.1723	0.0916	0.0593	0.0741
	α_1	0.3050	0.1891	**0.2562**	0.2495	0.2892	0.1859	**0.2558**	0.2480	0.0722	0.0375	0.0340	0.0312
	β_1	0.2124	0.2093	**0.3020**	0.2745	0.2067	0.2067	**0.3010**	0.2736	0.1119	0.0879	0.0759	0.0834
	itr			**6.8800**	5.4700			**7.0000**	5.0000			1.5128	1.3887
	nfs			**232.64**	233.81			**232.00**	235.50			17.554	16.255
0.25	ARL	5699.1	583.52	**236.91**	254.60	5400.7	583.68	**233.65**	256.01	1820.1	131.28	34.607	36.713
	ω	1.5545	0.8926	0.6337	**0.6041**	1.5358	0.8797	0.6149	**0.5939**	0.2771	0.2480	0.1277	0.1330
	α_1	0.2054	0.1102	0.2092	**0.2067**	0.2046	0.1079	0.2131	**0.2038**	0.0529	0.0323	0.0401	0.0373
	β_1	0.2971	0.3616	0.2671	**0.3078**	0.2953	0.3671	0.2801	**0.3251**	0.1068	0.1531	0.1131	0.1195
	itr			9.5900	8.0400			9.0000	8.0000			1.7928	1.9588
	nfs			422.70	414.63			418.00	416.50			24.641	24.726

The results of Table 2 show that when there is not nuisance in samples, RME method performs well and MLE method is the second proper method. This confirms the weakness of the filtering procedure in the absence of the outliers. When r increases to 0.02, FRME and RME methods are so closed in performance. In this situation, the robust procedure could well tackle the nuisance in both RME and FRME methods. Afterwards, when r becomes equal to 0.05, FRME and FMLE are the first and second well methods. This shows the replacement of robust procedure with filtering procedure in performance. Then, in the rate of 0.1 and 0.25, FMLE and FRME are the first and second ranked methods. This confirms the obtained results in the previous table which the proposed filtering procedure performs better than the other methods in dealing with large size of outliers in samples.

4-2- Example II

In this example, MLE, RME, FMLE and FRME methods are applied to estimate GARCH(1,1) parameters in the presence of different nuisance rates. The confidence level in the filtering procedure is set equal to 99.73 percent. Then, the performance of the residual Shewhart control chart with estimated parameters is evaluated based on in-control ARL criterion. The control limits (UCL and LCL) for the residual Shewhart control chart in Phase II is set equal to ±3 for predefined in-control ARL of 370. Table 3 shows mean, median and standard deviation of the in-control ARL, the estimated parameters of the GARCH model, the average number of filtering iterations and the average number of filtered samples when the size of nuisance (S) is set equal to 1.5. The highlighted columns in this table mark the first two best methods and the bold data indicate the best results.

Table 3. Descriptive statistics of the results when S is set equal to 1.5 for the second example

Statistic		Mean				Median				Standard Deviation			
Method		MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME
r	Criteria												
0	ARL	**370.03**	368.47	330.26	351.00	365.39	**368.68**	328.60	350.71	36.727	35.944	34.040	33.335
	ω	0.4083	**0.4038**	0.3968	0.3974	0.4051	**0.4027**	0.3946	0.3968	0.0365	0.0334	0.0397	0.0359
	α_1	0.2982	**0.2980**	0.2919	0.2944	0.2982	**0.3002**	0.2906	0.2945	0.0257	0.0235	0.0268	0.0236
	β_1	0.2931	**0.2969**	0.2946	0.3004	0.2934	**0.3013**	0.2957	0.3012	0.0430	0.0427	0.0488	0.0461
	itr			3.3700	2.8300			3.0000	3.0000			1.2685	1.2477
	nfs			19.000	17.810			18.000	17.000			5.4772	5.1673
0.02	ARL	404.15	385.08	351.68	**368.48**	398.49	388.06	346.54	**368.39**	47.508	39.351	39.727	36.596
	ω	0.4121	0.4069	0.3984	**0.4026**	0.4162	0.3999	0.3984	**0.4003**	0.0317	0.0470	0.0338	0.0470
	α_1	0.2990	0.2900	0.2926	**0.2887**	0.2995	0.2891	0.2921	**0.2862**	0.0234	0.0273	0.0217	0.0274
	β_1	0.2995	0.3062	0.2998	**0.3059**	0.2971	0.3041	0.2992	**0.3037**	0.0398	0.0573	0.0447	0.0581
	itr			3.1400	**2.6400**			3.0000	**3.0000**			1.3182	0.8471
	nfs			21.260	**19.970**			21.000	**20.000**			5.9876	5.1354
0.05	ARL	464.07	430.78	**381.65**	409.08	459.20	429.56	**378.78**	404.37	50.986	46.756	41.447	45.881
	ω	0.4336	0.4263	**0.4140**	0.4165	0.4309	0.4258	**0.4088**	0.4177	0.0450	0.0370	0.0427	0.0391
	α_1	0.2962	0.2972	**0.2864**	0.2974	0.2986	0.2980	**0.2869**	0.2968	0.0252	0.0285	0.0261	0.0294
	β_1	0.2973	0.2940	**0.2997**	0.2977	0.2979	0.2928	**0.3039**	0.2965	0.0491	0.0485	0.0522	0.0522
	itr			**3.3500**	2.8700			**3.0000**	3.0000			1.2822	1.1517
	nfs			**26.410**	25.500			**25.000**	25.000			6.3566	6.1669
0.1	ARL	562.93	499.83	**434.87**	474.27	561.35	498.46	**433.28**	475.51	68.548	59.004	51.958	55.387
	ω	0.4631	0.4551	**0.4305**	0.4449	0.4585	0.4545	**0.4240**	0.4413	0.0457	0.0474	0.0471	0.0482
	α_1	0.2979	0.2893	**0.2865**	0.2922	0.2969	0.2895	**0.2905**	0.2919	0.0263	0.0290	0.0272	0.0305
	β_1	0.2909	0.2903	**0.2993**	0.2917	0.2964	0.2860	**0.3097**	0.2881	0.0503	0.0520	0.0549	0.0555
	itr			**3.4700**	3.0100			**3.0000**	3.0000			1.0867	1.0683
	nfs			**32.770**	30.940			**33.000**	31.000			7.7926	6.7432
0.25	ARL	1027.4	858.24	**698.32**	796.90	1013.3	856.35	**696.69**	785.15	143.98	122.98	91.630	111.87
	ω	0.5516	0.5270	**0.5077**	0.5086	0.5417	0.5214	**0.5049**	0.5020	0.0635	0.0593	0.0649	0.0627
	α_1	0.2898	0.2746	**0.2710**	0.2777	0.2933	0.2783	**0.2698**	0.2776	0.0295	0.0290	0.0306	0.0288
	β_1	0.2921	0.3035	**0.2984**	0.3083	0.2988	0.3079	**0.2975**	0.3214	0.0581	0.0592	0.0646	0.0631
	itr			**4.0100**	2.9300			**4.0000**	3.0000			1.3521	0.9771
	nfs			**44.490**	41.260			**45.000**	40.000			7.3409	7.8491

The results of Table 3 show that when there is not nuisance in samples, RME method performs well and MLE method is the second proper method. When r increases to 0.02, FRME and RME performance is so closed. Afterwards, when r becomes equal or greater than 0.05, FMLE and FRME are the first and second well methods. This confirms the replacement of the robust procedure by the filtering procedure in performance as well as the result in the first example. Moreover, when the confidence interval widened, the performance of the filtering procedure dominate the robust procedure under lower levels of r when the size of nuisance is moderate (1.5). Table 4 shows the same results when the size of nuisance (S) is set equal to 3.

Table 4. Descriptive statistics of the results when *S* is set equal to 3 for the second example

Statistic		Mean				Median				Standard Deviation			
Method		MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME	MLE	RME	FMLE	FRME
r	Criteria												
0	ARL	**370.03**	368.47	330.26	351.00	365.39	**368.68**	328.60	350.71	36.727	35.944	34.040	33.335
	ω	0.4083	**0.4038**	0.3968	0.3974	0.4051	**0.4027**	0.3946	0.3968	0.0365	0.0334	0.0397	0.0359
	α_1	0.2982	**0.2980**	0.2919	0.2944	0.2982	**0.3002**	0.2906	0.2945	0.0257	0.0235	0.0268	0.0236
	β_1	0.2931	**0.2969**	0.2946	0.3004	0.2934	**0.3013**	0.2957	0.3012	0.0430	0.0427	0.0488	0.0461
	itr			3.3700	2.8300			3.0000	3.0000			1.2685	1.2477
	nfs			19.000	17.810			18.000	17.000			5.4772	5.1673
0.02	ARL	703.82	391.96	**360.61**	387.56	698.12	387.35	**359.73**	382.26	113.50	46.056	41.968	45.432
	ω	0.5079	0.4542	**0.4116**	0.4157	0.5046	0.4513	**0.4116**	0.4070	0.0728	0.0461	0.0451	0.0423
	α_1	0.3123	0.2795	**0.2870**	0.2924	0.3069	0.2793	**0.2856**	0.2912	0.0452	0.0241	0.0237	0.0235
	β_1	0.2679	0.2657	**0.2938**	0.2949	0.2615	0.2627	**0.2922**	0.2944	0.0833	0.0522	0.0525	0.0516
	itr			**3.7500**	2.9100			**3.5000**	3.0000			1.2092	0.9545
	nfs			**48.910**	47.570			**50.000**	48.000			7.9583	8.6389
0.05	ARL	1764.5	442.04	**428.85**	454.16	1744.6	440.71	**429.76**	452.26	412.24	69.481	46.673	60.704
	ω	0.6938	0.5413	**0.4423**	0.4600	0.6980	0.5435	**0.4472**	0.4558	0.0953	0.0608	0.0490	0.0504
	α_1	0.3245	0.2530	**0.2846**	0.2908	0.3190	0.2534	**0.2823**	0.2918	0.0611	0.0303	0.0280	0.0297
	β_1	0.2232	0.2181	**0.2868**	0.2714	0.2229	0.2159	**0.2809**	0.2752	0.0816	0.0655	0.0577	0.0581
	itr			**4.5400**	3.1500			**4.0000**	3.0000			1.2825	1.2503
	nfs			**91.180**	91.920			**91.000**	90.500			10.026	11.571
0.1	ARL	5557.5	**531.82**	593.65	596.13	5608.4	**526.77**	587.59	592.95	1842.6	88.517	88.209	74.790
	ω	0.9363	0.6285	**0.4950**	0.5080	0.9345	0.6238	**0.4914**	0.4978	0.1573	0.0950	0.0691	0.0658
	α_1	0.3072	0.1869	**0.2710**	0.2557	0.3072	0.1832	**0.2685**	0.2559	0.0731	0.0376	0.0339	0.0308
	β_1	0.2264	0.2385	**0.2891**	0.2906	0.2235	0.2285	**0.2842**	0.2934	0.1114	0.0935	0.0730	0.0655
	itr			**5.2400**	3.8300			**5.0000**	4.0000			1.2722	1.0736
	nfs			**153.16**	157.83			**154.00**	158.50			12.602	12.900
0.25	ARL	30424	1726.1	**1826.4**	1844.0	23151	**1622.7**	1705.9	1775.3	20183	471.15	549.20	445.19
	ω	1.5657	0.8936	**0.7947**	0.7769	1.5859	0.9092	**0.7801**	0.7406	0.3455	0.2279	0.1886	0.1962
	α_1	0.2150	0.1119	**0.1871**	0.1805	0.2045	0.1078	**0.1839**	0.1796	0.0666	0.0355	0.0480	0.0402
	β_1	0.2939	0.3567	**0.3033**	0.3248	0.2746	0.3391	**0.2909**	0.3228	0.1402	0.1436	0.1324	0.1300
	itr			**8.7115**	6.1400			8.5000	6.0000			2.0033	1.6019
	nfs			**260.48**	258.10			264.00	254.50			22.920	19.869

The results of Table 4 show that when there is not nuisance in samples, RME method performs well and MLE method is the second proper method. When *r* becomes equal or greater than 0.02, FMLE and FRME are the first and second well methods. This confirms the replacement of the robust procedure by the filtering procedure under lower levels of *r* when the size of nuisance (*S*) is rather high (equal to 3).

5- Foreign exchange rate

The main motivation of this research is monitoring the USD/IRR exchange rate. The effect of foreign exchange rate on the economy is vast and obvious (Mankiw, 2014). The effect of currency fluctuations on the international trades forced decision makers to investigate changes in exchange rate (Oskooee and Hegerty, 2007). For example, a country with high volatile currency could face with less foreign investments. The control chart can help practitioners in detecting any atypical changes in foreign exchange rate. If these diagnostics are followed by appropriate decisions, it can lead to a less volatile process based on the concept of six sigma (Frisen, 2008). The USD/IRR exchange rate is also well studied in the literature of the econometrics (Norouzzadeh and Rahmani, 2006). There are different approaches for modeling the USD/IRR exchange rate (Fahimifard et al. 2009). To the best of our knowledge, there is no paper for monitoring the USD/IRR exchange rate by using control charts. The most promising model in this subject is GARCH model (Araghi and Pak, 2013). Therefore, in this section, the performance of the proposed methods is illustrated through a real case study.

The data set, available upon request, consists of 3279 daily observations from the first working day of the year 1384 Solar Hijri (S.H.) until 13th days of the month Mordad from the year of 1395 S.H. In financial analyses, it is usual to transform samples in logarithm of daily returns as Equation (14). Therefore, let y_t denotes the original observation, then, x_t reports the daily log return. Figure 1 shows the original and daily log returns of USD/IRR exchange rate.

$$x_t = 100 \log\left(\frac{y_t}{y_{t-1}}\right). \tag{14}$$

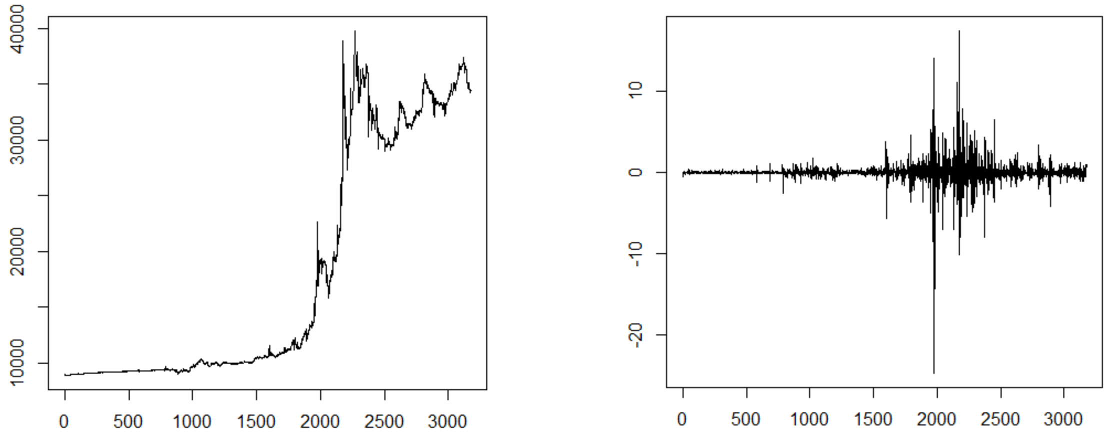

Figure 1. The original and daily log returns of USD/IRR exchange rate

Figures 2 and 3 respectively show the autocorrelation and partial autocorrelation of the first and second order of log returns based on autocorrelation function (ACF) and partial ACF (PACF). As it can be seen, there are significant autocorrelations especially in the second order of log returns. In such conditions, the GARCH model can well define the process behavior.

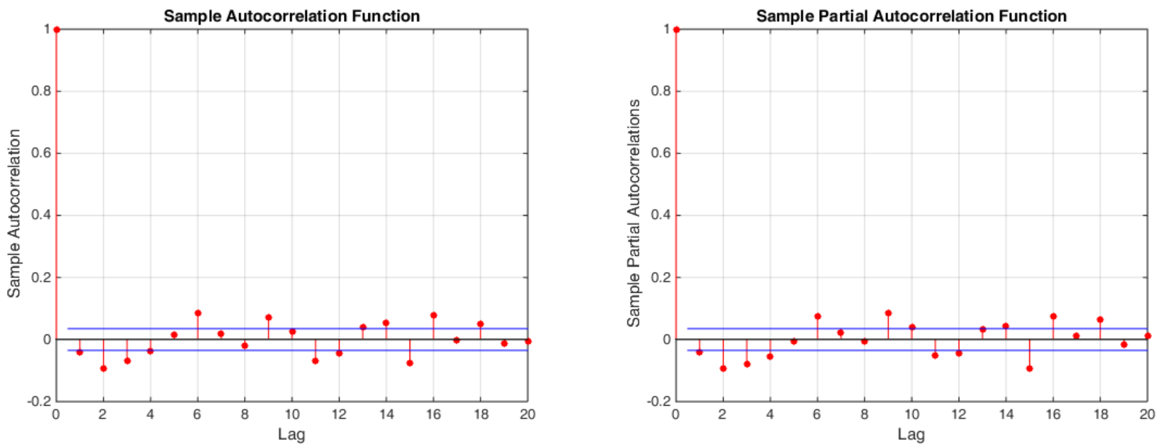

Figure 2. ACF and PACF of the first order of log returns

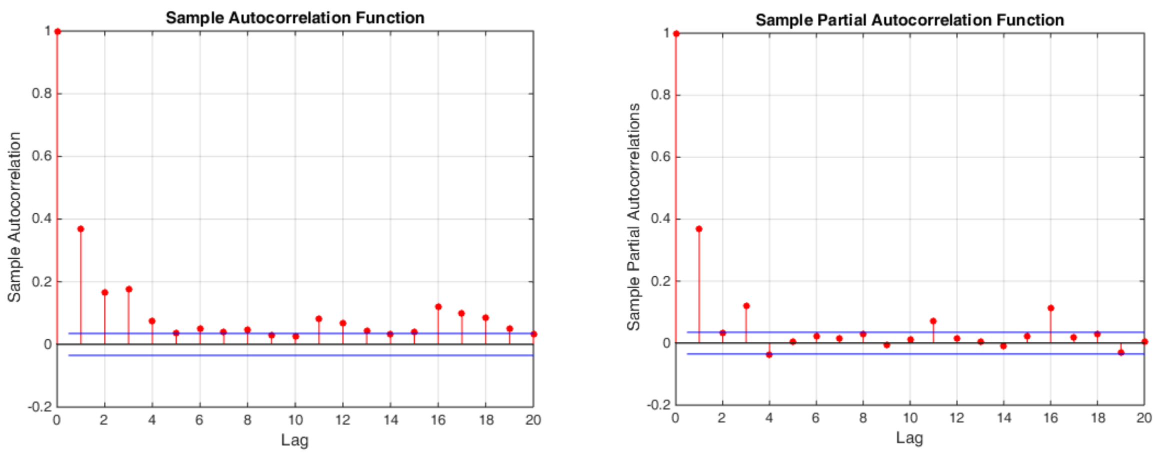

Figure 3. ACF and PACF of the second order of log returns

The samples of the first 9 years including 2594 observations are selected for phase I analyses. The following 685 samples are considered for monitoring purpose in Phase II. The augmented Dickey-Fuller test rejects the null hypothesis of unit root in log returns. Then, the parameters of the model are estimated based on MLE, RME, FMLE and FRME methods. Note that the confidence level in filtering procedure is set equal to 99.17 percent. The results of residual analyses confirm that the model is sufficient. As instance, the residual analyses of MLE method are reported. The similar outcomes are obtained for the other methods as well. Figures 4 and 5 show the autocorrelation and partial autocorrelation of the first and second orders of the residuals based on ACF and PACF, respectively.

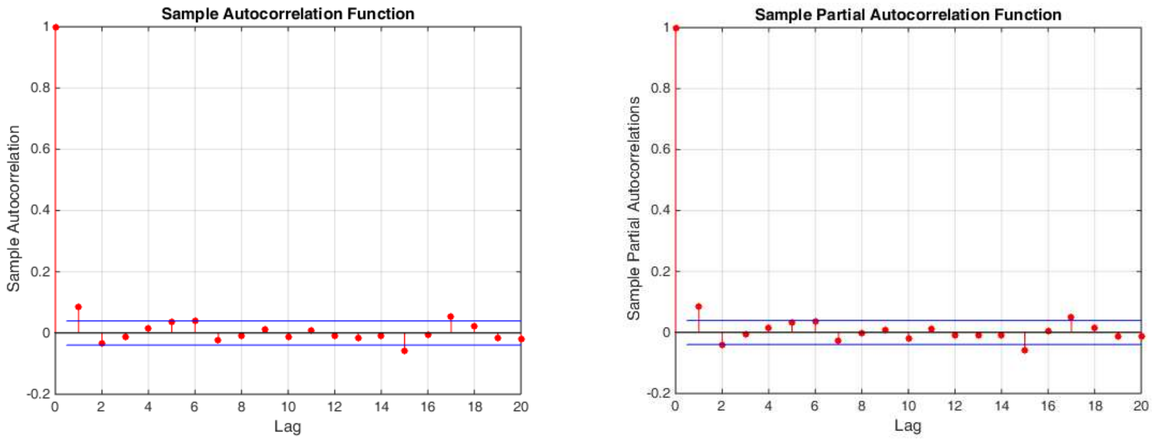

Figure 4. ACF and PACF of the first order of the residuals in MLE method

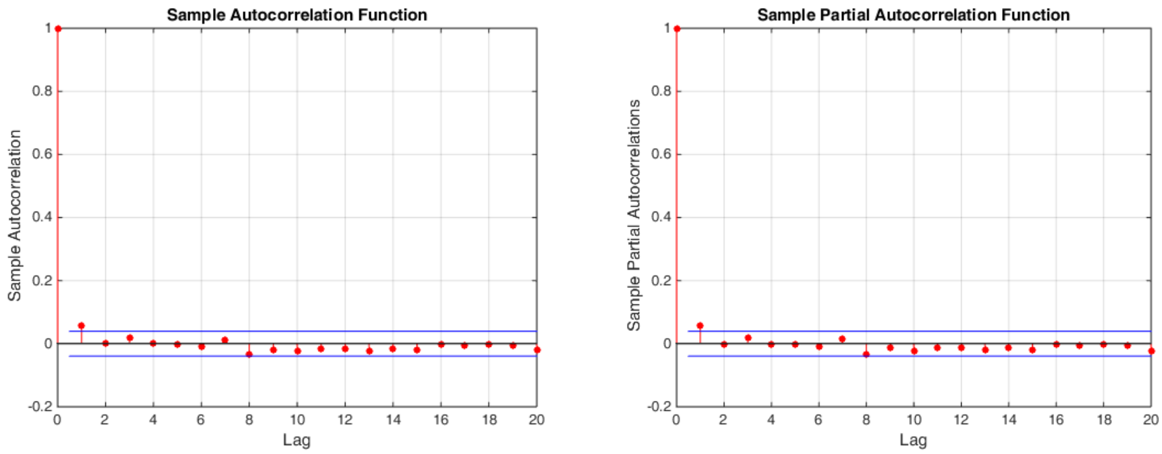

Figure 5. ACF and PACF of the second order of the residuals in MLE method

As shown in Figures 4 and 5, the autocorrelations in both orders are eliminated. The average of the residuals is equal to 0.0854 which is so close to zero. Although, the normality assumption of the residuals is rejected, as shown in Figure 6 the histogram of the residuals is so close to the normal distribution. The stability of the variance over time can be seen in this figure as well.

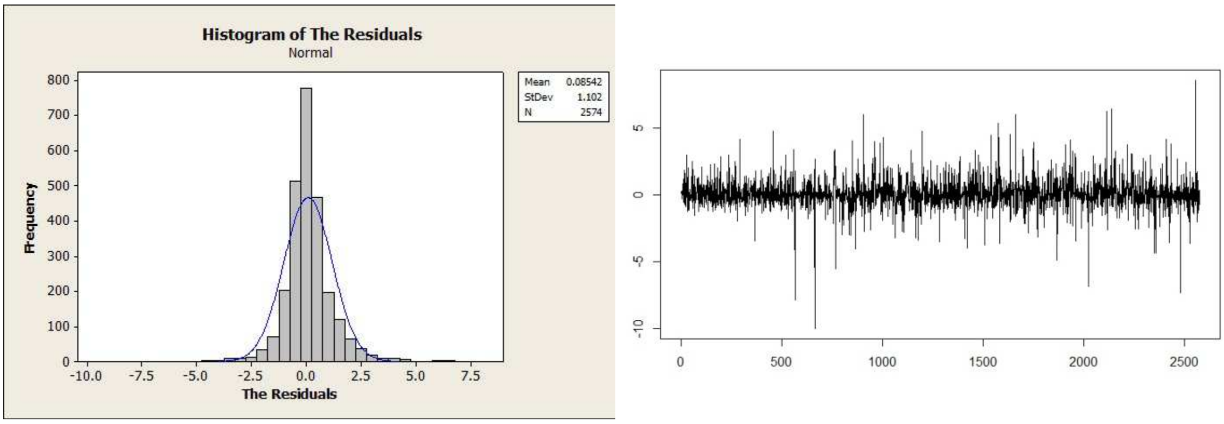

Figure 6. Histogram and trends of the residuals in MLE method

Table 5 shows the estimated parameters in all methods. The results of this table show that the numbers of filtered samples in FMLE and FRME methods are 363 and 400, respectively. The percent of filtered samples respectively are 14% and 15% for FMLE and FRME methods which are rather great respect to 2594 sample data. This demonstrates the existence of outliers in sample data. Therefore, it is expected that the robust or filtering approach performs better than the traditional MLE method. The analyses of the values of the estimated parameters indicate significant difference between the estimators. In the other words, adding filtering and robust procedures to the estimation method results in more accurate and precise parameters values. For example, the estimated value of ARCH parameter (α_1) in MLE, FMLE, RME and FRME methods are respectively increasing from 0.1503 to 0.3159 which is more significant. Hence, FRME method is recommended to design residual Shewhart control chart in Phase II.

Table 5. The estimation results for the real example

Method	MLE	RME	FMLE	FRME
ω	0.0006	0.0009	0.0002	0.0009
α_1	0.1503	0.2676	0.1999	0.3159
β_1	0.8497	0.6892	0.8001	0.6638
itr			24	36
nfs			363	400

Afterwards, the residual Shewhart control chart is designed with the estimated parameters based on FRME method for monitoring the rest of the samples. The control limits are set equal to ±2.6383 for the in-control ARL equals to 120. Figure 7 shows the control limits as well as the control statistics over time.

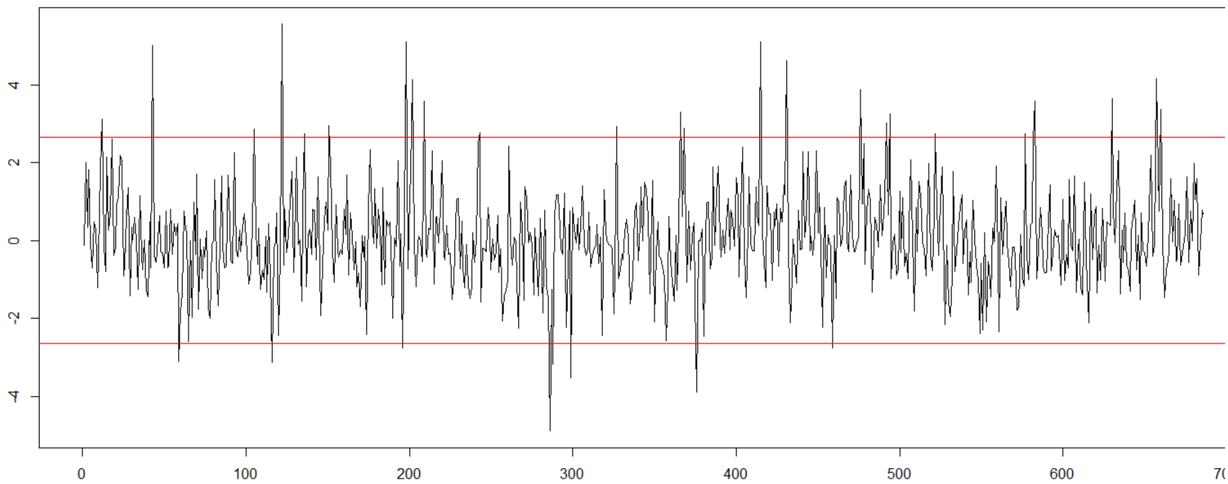

Figure 7. Control statistics in Phase II with the estimated parameters based on FRME method

After each signal, the out-of-control state could be interpreted based on the market analyses. These signals could be useful for decision makers. Moreover, if these decisions are followed by real actions in the market, they can bring less volatility for the future trends of the market. However, according to the results of the simulation studies, it is expected that Phase II control chart with estimated parameters based on MLE method is less sensitive. Table 6 shows the number of signals given by Phase II control chart based on different estimation methods.

Table 6. The number of control chart signals in the real example

Estimation Method	MLE	RME	FMLE	FRME
The number of signals	17	34	21	32

As it is expected, MLE method is insensitive to changes in process. This is confirmed by the obtained results in the real example. Figure 8 shows the control statistics with the estimated parameters based on MLE method over time as well as the corresponding control limits.

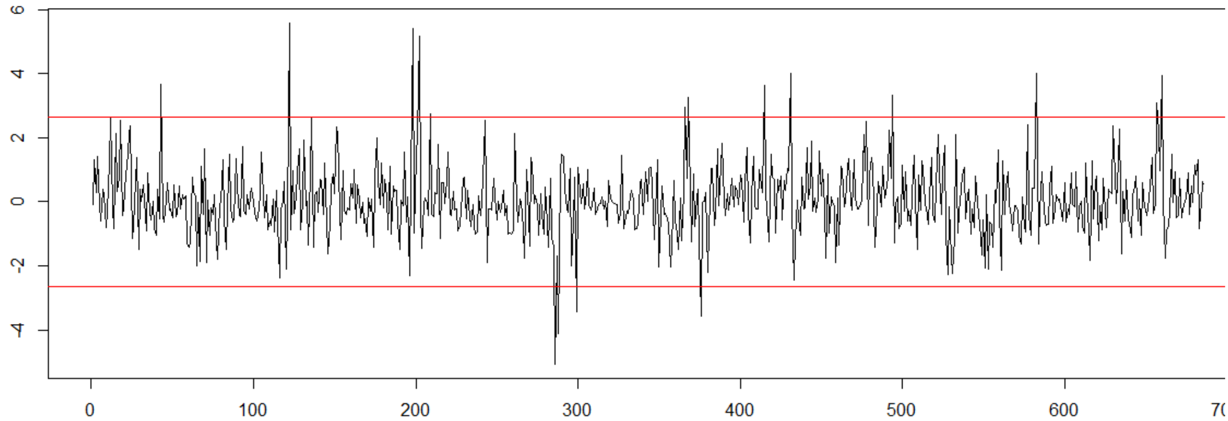

Figure 8. Control statistics in Phase II with the estimated parameters based on MLE method

5- Conclusions and future research

In this paper, the performance of residual Shewhart control chart with the estimated parameters was evaluated for monitoring financial GARCH process in the presence of outliers. The reason for selecting GARCH model was the generality of the model for describing the financial process behavior. Moreover, this model could well define USD/IRR exchange rate as the main motivation of this research. To estimate the parameters of the model, in addition to the traditional MLE method, some robust methods were proposed to handle the outliers. Simulation studies revealed that the control chart was significantly affected by the outliers. Generally, the outliers made the in-control ARL greater than the predefined expected value. In other words, the control chart was insensitive to the changes in the process. In different numerical examples, the control chart based on RME and MLE methods performed better than the others when there is only clean historical data. While by increasing the rate of the outliers, the control chart based on FMLE and FRME methods resulted in slightly better in-control ARL performance. Finally, the proposed methods were applied for monitoring USD/IRR exchange rate. In the real example, the control chart was more sensitive when the robust methods were applied in estimation procedure. This confirmed the obtained results in simulation studies. The performance of control charts in monitoring financial processes in the presence of outliers can be investigated in future research.

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